Varianta 76

Subiectul I

a) z=-5i

b) $\sqrt{2}+\sqrt{13}+\sqrt{17}$

c) $|\vec{v}|=\sqrt{29}$

d) $m_1 \cdot m_2 = -1 \Rightarrow \frac{1}{2}\cdot\frac{2}{a} = -1 \Rightarrow a = 1.$

e) $V = 9$

f) $a = -\frac{9}{26}; b = -\frac{7}{26.}$

Subiectul II

1. a) $\sqrt{1}\sqrt{2}...\sqrt{n} = \sqrt{n!} < 5 \Leftrightarrow n \in \{1,2,3,4\}$

b) 3 divide pe 3,6,9, iar 2 divide pe 2,4,6,8,10 $\Rightarrow$ probabilitatea este $\frac{7}{10}$.

c) $\{a, b\}$ este continuta neaparat in multimile cautate, dar c, d, e pot fi din acestea sau nu $\Rightarrow$ exista $2^3 = 8$ submultimi cautate.

d) $5^x = 25 \Leftrightarrow x = 2.$

e) $x^2 - ax + 9 > 0 \Leftrightarrow \Delta = a^2 - 36 < 0 \Leftrightarrow a \in (-6,6)$

2. a) $f'(x) = 2006x^{2005} + 1$

b) Daca x este un punct de extrem, atunci $f'(x) = 0 \Rightarrow x^{2005} = \frac{-1}{2006} \Rightarrow x = \sqrt[2005]{\frac{-1}{2006}}$

c) $f''(x) = 2006 \cdot 2005x^{2004} \geq 0, (\forall)x \in \mathbf{R} \Rightarrow f$ este convexa pe $\mathbf{R}$.

d) $\lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = f'(1) = 2007$

e) $\int_0^1 f(x)dx = \left(\frac{x^{2007}}{2007} + \frac{x^2}{2}\right)\Bigg|_0^1 = \frac{2009}{4014}.$

Subiectul III

a) $Q(x) = x - 1, R(x) = 0.$b) Avem $a + b = 1, a \cdot b = -1$ si $(x^2 + ax + 1)(x^2 + bx + 1) = x^4 + (a+b)x^3 + (ab+2)x^2 + (a+b)x + 1.$

c) Singura descompunere a *lui* g in **R[X]** este cea de la b) , descompunere in care polinoamele $(x^2 + ax + 1)$ si $(x^2 + bx + 1)$ nu sunt in **Q[X].**

d) Se efectueaza impartirea.e) Se verifica f) $A^5 = I_2 \Rightarrow (\det A)^5 = 1 \Rightarrow \det A = 1 \neq 0.$

g) $A^5 = I_2 \Leftrightarrow f(A) = 0.$ Daca notam $h = X^2 - (r+u)X - (ru - st)$ mai avem $h(A) = 0.$ Din d) $f = q \cdot h + r$ cu grad $r = 1$, deci $f(A) = q(A)h(A) + r(A).$ Rezulta $r(A) = 0 \Rightarrow A = \alpha I_2$ cu $\alpha \in \mathbf{Q}$ si din $A^5 = I_2 \Rightarrow \alpha^5 = 1$, deci $A^5 = I_2$

Subiectul IV

a) Folosim formula pentru progresie geometrica: $1+a+...+a^n = \frac{1-a^{n+1}}{1-a}$.

b) Luam in a), $a=-x^2$

c) $1+x^2 \geq 1$.

d) $\int_0^1 \frac{x^{2(n+1)}}{1+x^2}dx \leq \int_0^1 x^{2(n+1)}dx = \frac{1}{2n+3} > 0$

e) $\int_0^1 \frac{1}{1+x^2}dx = arctg\text{x}\,|_0^1 = \frac{\pi}{4}$

f) Integram relatia de la b) intre 0 si 1 si conform lui e) avem:

$$\frac{\pi}{4} = 1-\frac{1}{3}+\frac{1}{5}-...+(-1)^n\frac{1}{2n+1}+(-1)^{n+1}\int_0^1 \frac{x^{2(n+1)}}{1+x^2}dx.$$

Daca trecem la limita, din d) rezulta $\lim_{n->\infty} a_n = \frac{\pi}{4}$.

$$g) Avem\,\frac{\pi}{4} = a_{2n} - \int_0^1 \frac{x^{4n+2}}{1+x^2}dx => a_{2n} - \frac{\pi}{4} = \int_0^1 \frac{x^{4n+2}}{1+x^2}dx => n(a_{2n}-\frac{\pi}{4}) =$$

$$= n\int_0^1 \frac{x^{4n+2}}{1+x^2}dx = \frac{n}{4n+3}\int_0^1 (x^{4n+3})'\frac{1}{1+x^2}dx = \frac{n}{4n+3}(\frac{x^{4n+3}}{1+x^2})|_0^1 - \int_0^1 x^{4n+3}\frac{-2x}{(1+x^2)^2}dx) =$$

$$\frac{n}{4n+3}(\frac{1}{2}+2\int_0^1 \frac{x^{4n+4}}{(1+x^2)^2}dx)$$

Intrucat $\lim_{n->\infty}\int_0^1 \frac{x^{4n+2}}{(1+x^2)2}dx = 0 => \lim_{n->\infty} n(a_n - \frac{\pi}{4}) = \frac{1}{8}$.